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Study of baryons in a combination of large-N_c QCD and constituent approach

LORENZO CIMINO* – CLAUDE SEMAY† – CINTIA T. WILLEMYNS‡

UNIVERSITY OF MONS

* lorenzo.cimino@umons.ac.be † claude.semay@umons.ac.be ‡ cintia.willemyns@umons.ac.be

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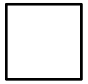
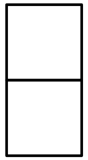
Brief recap of large-N_c QCD

- QCD is a gauge theory based on the colour group $SU(N_c = 3)$, where N_c is the number of colours
- At hadronic energy level, the strong coupling constant $\alpha_s \approx 1$
- In the limit $N_c \to \infty$, the strong interaction simplifies
  - Pertubative expansion in $1/N_c$ [1,2]

[1] G. 't Hooft. Nucl. Phys. B **72**, 461 (1974) [2] E. Witten. Nucl. Phys. B **160**, 57 (1979)

Brief recap of large-N_c QCD

- Different limits are possible. But …
  - Resulting theory must reduce to QCD at $N_c = 3$
  - Physical states are colour singlets
- ‘t Hooft limit ($\mathrm{QCD_F}$) : quarks stay in the fundamental representation 
  - Baryons are composed of $n_q = N_c$ quarks
- $\mathrm{QCD_{AS}}$: quarks are in the 2-indices antisymmetric representation 
  - Baryons are composed of $n_q = N_c(N_c - 1)/2$ quarks [3]
- $\mathrm{QCD_S}$, $\mathrm{QCD_{adj}}$, Corrigan-Ramond limit, etc.

[3] S. Bolognesi. Phys. Rev. D **75**, 065030 (2007)

Constituent approach

- The exchange of virtual gluons is replaced by a potential
  - Typically: funnel potential $V_{qq}(r) = \alpha r - \frac{\beta}{r}$
  - Relativistic $T(p) = \sqrt{p^2 + m^2}$ or non-relativistic $T(p) = m + p^2/2m$ kinematics

- At the end, we need to solve a Schrödinger-like equation

Combination of both approaches

- Immediate problem: one has to solve an Hamiltonian with $n_q(\to \infty)$ particles

$$H = \sum_{i=1}^{n_q} T_i(|\boldsymbol{p}_i|) + \sum_{i=1}^{n_q} U_i(|\boldsymbol{r}_i - \boldsymbol{R}|) + \sum_{i<j=2}^{n_q} V_{ij}(|\boldsymbol{r}_i - \boldsymbol{r}_j|),$$

  - Treat baryons as 3-body systems and add large-N_c results [4]
  - Use a method which can deal with large-n_q systems : envelope theory (ET)

[4] C.T. Willemyns, N.N. Soccola. Phys. Rev. D **98**, 034019 (2018)

Envelope theory

- **Practical guide** : the following n_q-body Hamiltonian of **identical** particles

$$H = \sum_{i=1}^{n_q} T(|\boldsymbol{p}_i|) + \sum_{i=1}^{n_q} U(|\boldsymbol{r}_i - \boldsymbol{R}|) + \sum_{i<j=2}^{n_q} V(|\boldsymbol{r}_i - \boldsymbol{r}_j|),$$

has an approximate spectrum E given by the following set of 3 equations [6]

$$E = n_q T(p_0) + n_q U\left(\frac{r_0}{n_q}\right) + C_{n_q} V\left(\frac{r_0}{\sqrt{C_{n_q}}}\right),$$

$$r_0 p_0 = Q,$$

$$n_q p_0 T'(p_0) = r_0 U'\left(\frac{r_0}{n_q}\right) + \sqrt{C_{n_q}}\, r_0 V'\left(\frac{r_0}{\sqrt{C_{n_q}}}\right).$$

- $p_0^2 = \langle \boldsymbol{p}_i^2 \rangle$ and $r_0^2 = N^2 \langle (\boldsymbol{r}_i - \boldsymbol{R})^2 \rangle \ \forall\, i,j$
- $Q = \sum_{i=1}^{n_q-1} (2n_i + l_i + D/2)$
- $C_{n_q} = n_q(n_q - 1)/2$

[6] C. Semay, C. Roland. Res. Phys. **3**, 231 (2013)

Envelope theory - Example

- We consider a Gaussian potential $V(r) = -V_0 e^{-r^2/R^2}$, $U(r) = 0$, with a non-relativistic kinematics $T(p) = p^2/2m$ [7,8]

$$r_0 p_0 = Q$$

$$n_q p_0 T'(p_0) = r_0 U'\left(\frac{r_0}{n_q}\right) + \sqrt{C_{n_q}}\, r_0 V'\left(\frac{r_0}{\sqrt{C_{n_q}}}\right)$$

$\longrightarrow$

$$r_0 = n_q^{1/2}(n_q - 1)^{1/2}\sqrt{-W_0(Y)}\,R$$

$$\text{with } Y = -\frac{1}{n_q^{1/2}(n_q-1)}\frac{Q}{R\sqrt{2mV_0}}$$

  - $W_0(Y)$ is the Lambert function

$$E = n_q T(p_0) + n_q U\left(\frac{r_0}{n_q}\right) + C_{n_q} V\left(\frac{r_0}{\sqrt{C_{n_q}}}\right)$$

$\longrightarrow$

$$E = -\frac{n_q(n_q-1)}{2} V_0 Y^2 \frac{1 + 2W_0(Y)}{W_0(Y)^2}$$

  - **The number of particles n_q can be arbitrary**

[7] C. Semay. Few-Body Syst. **56**, 149 (2015) [8] C. Semay, L. Cimino. Few-Body Syst. **60**, 64 (2020)

Envelope theory - Example

- We consider a Gaussian potential $V(r) = -V_0 e^{-r^2/R^2}$, $U(r) = 0$, with a non-relativistic kinematics $T(p) = p^2/2m$

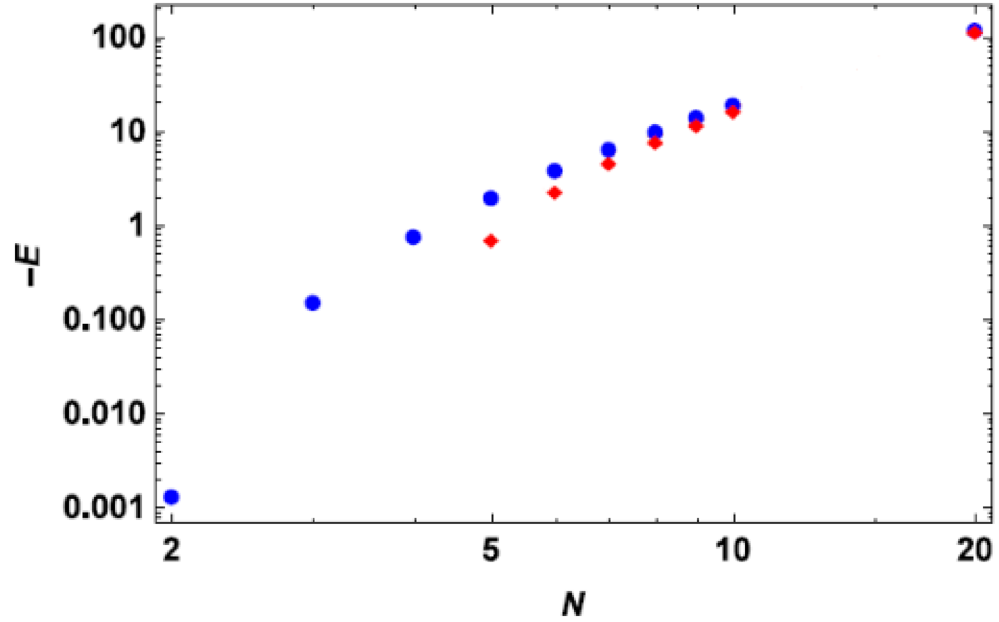


Figure 1: Bosonic ground state (GS) energy at $D = 3$ dimensions.
ET results [7] in red ,exact results [9] in blue.

[9] J. Horne, J. A. Salas, K. Varga. Few-Body Syst. **55**, 1245 (2014)

Hamiltonian - Description

- We will use the following Hamiltonian [10] to describe our large-N_c baryons

$$H_0 = \sum_{i=1}^{n_q} \sqrt{\boldsymbol{p}_i^2} + \frac{C_q}{C_\square} \sigma \sum_{i=1}^{n_q} |\boldsymbol{r}_i - \boldsymbol{R}| + \frac{1}{2}(C_{qq} - 2C_q) \frac{\alpha_0}{N_c} \sum_{i<j=2}^{n_q} \frac{1}{|\boldsymbol{r}_i - \boldsymbol{r}_j|}$$

Semi-relativistic kinematics $T(p) = \sqrt{\boldsymbol{p}^2 + m^2}$ with vanishing mass (u and d quarks)

Confinement from flux-tube model where σ is the string tension and $\boldsymbol{R}$ the centre of mass

Short-range potential from one-gluon exchange (OGE) process

- C_q, C_{qq} and $C_\square$ are quadratic Casimir operators of $SU(N_c)$
- σ and α_0 are constant with N_c

[10] F. Buisseret, C. Semay. Phys. Rev. D **82**, 056008 (2010)

Hamiltonian - Resolution

- Thanks to the ET, one can find analytical upper bounds of the Hamiltonian H_0 [10]

$$r_0 p_0 = Q$$

$$n_q p_0 T'(p_0) = r_0 U'\left(\frac{r_0}{n_q}\right) + \sqrt{C_{n_q}}\, r_0 V'\left(\frac{r_0}{\sqrt{C_{n_q}}}\right) \longrightarrow r_0 = \sqrt{\frac{C_\square}{C_q \sigma}\left(n_q Q + (C_{n_q}^2)^{3/2}\frac{1}{2}(C_{qq} - 2C_q)\frac{\alpha_0}{N_c}\right)}$$

$$E = n_q T(p_0) + n_q U\left(\frac{r_0}{n_q}\right) + C_{n_q} V\left(\frac{r_0}{\sqrt{C_{n_q}}}\right) \longrightarrow M_0 = \sqrt{\frac{C_q}{C_\square}\sigma\left(n_q Q + (C_{n_q}^2)^{3/2}\frac{1}{2}(C_{qq} - 2C_q)\frac{\alpha_0}{N_c}\right)}$$

- After computations of the Casimir operators, one finds for the GS [11]

$$M_0^{\mathrm{QCD_F}} = N_c\sqrt{\sigma\left(6 - \frac{\alpha_0}{\sqrt{2}}\right)} \qquad M_0^{\mathrm{QCD_{AS}}} = \frac{N_c^2}{2}\sqrt{\sigma\left(12 - 2\sqrt{2}\alpha_0\right)}$$

➢ **At dominant order**, GS baryons mass scales as N_c and N_c^2 in $\mathrm{QCD_F}$ and $\mathrm{QCD_{AS}}$

[10] F. Buisseret, C. Semay. Phys. Rev. D **82**, 056008 (2010)

Correction to the mass

- Up to now, we have looked only at the dominant contribution to the mass. Several corrections are possible:
  - Presence of strange quarks (with $m_s \neq 0$)
  - Spin interactions

Correction to the mass

- Up to now, we have looked only at the dominant contribution to the mass. Several corrections are possible:
  - **Presence of strange quarks (with $\boldsymbol{m_s \neq 0}$)**
  - Spin interactions

- If we add $n_s \ll n_q$ strange quarks with a mass $m_s \ll \langle\sqrt{\boldsymbol{p}^2}\rangle$ then, from a **perturbation** of the ET, we find a correction [10]

$$\Delta M^{\mathrm{QCD_F}} \approx n_s \frac{m_s^2}{6\sqrt{\sigma}} \sqrt{6 - \frac{\alpha_0}{\sqrt{2}}} \qquad \Delta M^{\mathrm{QCD_{AS}}} \approx n_s \frac{m_s^2}{6\sqrt{\sigma}} \sqrt{3 - \frac{\alpha_0}{\sqrt{2}}}$$

  - Proportional to m_s^2 ($SU(3)_F$ symmetry breaking parameter) and of order $O(1)$

[10] F. Buisseret, C. Semay. Phys. Rev. D **82**, 056008 (2010)

Correction to the mass

- Up to now, we have looked only at the dominant contribution to the mass. Several corrections are possible:
  - Presence of strange quarks (with $m_s \neq 0$)
  - **Spin interactions**
- Spin interactions are in the form of 2-body forces $W_{ij} \sim \vec{s}_i \, . \vec{s}_j$
  - Unfortunately, the ET cannot treat such interactions
- We know from large-N_c that spin corrections scale as $1/n_q$
  - Treat the force as a **perturbation** $\langle W_{ij} \rangle = \langle \phi | W_{ij} | \phi \rangle$
  - $|\phi\rangle$ is an eigenvector of the spin-independant Hamiltonian H_0

Eigenvector of spin-independent H

- We will write $|\phi\rangle$ as [11]

anti-symmetric

symmetric

$$|\phi\rangle = |\phi_C\rangle\, |\phi_X\rangle\, |\phi_{\mathrm{SF}}\rangle$$

Colour part : *anti-symmetric*

Spin – flavour part : *symmetric*

Space part: approximated by a set of harmonic oscillators (ET)

$$|\phi_X\rangle = \prod_{i=1}^{n_q-1} |n_i, l_i, \lambda_i, \boldsymbol{x}_i\rangle$$

The ground state, $n_i = l_i = 0$, is *symmetric*

[11] F. Buisseret, N. Matagne, C. Semay. Phys. Rev. D **85**, 036010 (2012)

Eigenvector of spin-independent H

- We will write $|\phi\rangle$ as [11]

anti-symmetric

symmetric

$$|\phi\rangle = |\phi_C\rangle\, \underbrace{|\phi_X\rangle\, |\phi_{\rm SF}\rangle}$$

Colour part : *anti-symmetric*

➢ Since $|\phi_X\rangle|\phi_{\rm SF}\rangle$ is completely symmetrised

$$\langle W\rangle = \sum_{i<j=2}^{n_q} \langle W_{ij}\rangle = C_{n_q}\,\langle W_{12}\rangle$$

[11] F. Buisseret, N. Matagne, C. Semay. Phys. Rev. D **85**, 036010 (2012)

Spin correction

- The spin-spin interaction comes from OGE process

$$W_{12}^{\mathrm{OGE}} = -\frac{8\pi}{3}\frac{1}{m^2}\frac{\alpha_0}{N_c}\delta^3(\boldsymbol{r}_{12})\frac{1}{2}(C_{qq} - 2C_q)\boldsymbol{s}_1 \cdot \boldsymbol{s}_2$$

  - $1/m^2$ comes from a non-relativistic approach. We should replace it by the operator $1/(\boldsymbol{p}^2 + m^2)$ [12]
  - In the framework of the ET, we approximate this operator by $\frac{1}{p_0^2}$ $(m = 0)$

- Computations of mean values in the GS [11]

$$\langle \phi_X(GS)|\delta^3(\boldsymbol{r}_{12})|\phi_X(GS)\rangle = \langle 0,0,\lambda_1,\boldsymbol{x}_1|\delta^3(\boldsymbol{r}_{12})|0,0,\lambda_1,\boldsymbol{x}_1\rangle = \frac{\lambda_1^3}{\pi^{3/2}} \qquad \langle \boldsymbol{s}_1 \cdot \boldsymbol{s}_2\rangle = \frac{S(S+1) - 3n_q}{8C_{n_q}^2}$$

[12] W. Lucha, F. F. Schöberl, D. Gromes. Phys. Rep. **200**, 127 (1991)

Spin correction

- Putting all results together [11]

$$\langle W_{\mathrm{F}}^{\mathrm{OGE}} \rangle = \frac{\alpha_0 A}{2\pi^{3/2}} \sqrt{\frac{\sigma}{6(12-\sqrt{2}\alpha_0)}} \left[\frac{S(S+1)}{N_c} - \frac{3}{4} \right] + O(\frac{1}{N_c}),$$
$$\langle W_{\mathrm{AS}}^{\mathrm{OGE}} \rangle = \frac{\alpha_0 A}{\pi^{3/2}} \sqrt{\frac{\sigma}{6(6-\sqrt{2}\alpha_0)}} \left[\frac{S(S+1)}{N_c^2/2} - \frac{3}{4} \right] + O(\frac{1}{N_c}).$$

  - In all limits $\langle W^{\mathrm{OGE}} \rangle \propto S(S+1)/n_q$
  - Spin-independent terms are present (unavoidable in potential model)

[11] F. Buisseret, N. Matagne, C. Semay. Phys. Rev. D **85**, 036010 (2012)

Outlooks

- The envelope theory is of great interest to study baryons in a combination of large-N_c QCD and constituent approach [10,11]
- Recently, the ET has been generalised to treat systems with N identical particles plus a different one [13,14]. It opens new possibilities
  - Baryons with heavy quarks
  - **Hybrid baryons** : composed of 3 quarks and a constituent gluon (or N quarks and a constituent gluon in large-N_c)
- Future works:
  - Construction of the Hamiltonian
  - Construction of the eigenvectors (colour, space and spin-flavour part)
  - Difficulties : gluons have helicity quantum number

[13] C. Semay, L. Cimino, C. Willemyns. Few-Body Syst. 61, 19 (2020) [14] L. Cimino, C. Semay. Braz. J. Phys. 52, 45 (2022)

Thank you for your attention